Development and study of a multi-criteria optimal control algorithm for the static mode of the fruit tunnel dehydration process

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Summary
In the study, based on the analysis of the technological process of drying apricots on a tunnel-type drying machine, the governing parameters and the control parameters at the exit of dehydrator as well as disturbing factors were determined. Fruit and air control parameters at the outlet of the tunnel-type drying chamber were determined as the final value of the state vector components, the solution of the system of non-linear ordinary differential equations with distributed parameters, describing the static mode of the drying process. With respect to assessing the quality of drying process control, the requirements of minimum energy consumption during drying, achieving maximum productivity and maintaining the desired value of the residual moisture content of the product were used. In order to reach a compromise between these requirements, a Pareto-optimal solution of the problem was proposed, for which a linear fold of the first and second criteria was introduced and the dependence of the minimum value of a new criterion on the fold parameter was considered. Based on the analysis of the results, approximately 5 % reduction in energy consumption with a significant increase in the quality of the dried fruits could be achieved.

Keywords
tunnel dehydrator; fruit; optimal control; static mode; Pareto-optimal solution; energy minimization; maximum productivity

A drying process is a form of food processing and preservation aimed at prolonging the shelf life of some food products while minimizing volume and weight during transportation. Dehydration reduces water activity and increases concentration of sugars in raw materials, which prevents enzymatic degradation and inhibits microbial growth. Therefore, storage duration of dried fruits is long and so they can be available for a long time. Due to the almost total absence of water in the dried product, most spoilage and microbiological reactions are stopped, which ultimately leads to a high quality product. However, if the level of moisture content drops too low, a fruit may become hard, unsuitable for eating and require rehydration. As a result of research, it has been established that fruits should be dehydrated to a certain optimum value of moisture in order to maintain their quality for further processing or storage, otherwise we would deal with unjustified costs, because when overdrying fruits, energy consumption increases and product quality decreases. Dried fruits should have a moisture content that significantly reduces the possibility of the growth of yeasts and filamentous fungi. The residual moisture content of the product is a major technological parameter of the drying process, which determines the quality of the product. In the food industry, a certain portion of the energy consumed falls on drying the product and energy saving reduces the cost of the product. Moreover, it determines to a great extent the technical-economic efficiency of the drying process. Based on these circumstances, the effective solution of the problem of optimal control of the drying process using modern computer technologies is a highly topical issue [1–5].

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Fruits have been dried under natural conditions since ancient times. The traditional drying method requires simple equipment, require little energy and economic costs, but the quality of the dried product is too low compared to industrial drying methods. Among the industrial drying methods, the convection drying method, which is carried out in devices of various types and designs, has found wide application in fruits drying, in accordance with the requirement of ensuring the preservation of the fruit structure and other properties. Since the main technological parameters of the drying process are of great importance in determining the quality of the product, it makes sense to analyse the influence of the optimized parameters on the colour of the dried fruits together with content of β-carotene and vitamin C in them [3, 4].

The research results of Elwakeel et al. [4] showed that heat treatment of dates up to the maximum temperature of 60–65 °C, at a certain rate of air flow, has a positive effect, which ensures the appropriate level of moisture and quality of date sugar. Golisz et al. [5] and Cosme-De Vera et al. [6] studied the fruits drying process with horizontal hot air flow in laboratory drying machines and established that the drying rate was influenced by the temperature of the drying agent more than by the air blowing rate in the dryer. In the study, a mathematical model of the second period of the drying process of fruits was developed, based on which, as a result of the optimization, it was established that in order to increase the quality and shelf-life of the product, it is advisable to maintain a low temperature of the drying agent at the beginning and at the end of the drying process.

Lapczynska-Kordon et al. [7] carried out multi-criteria optimization of the change in quality indicators (taste, colour, odour) of dried fruits obtained by microwave convection drying of apples. The optimal decision was made by the method of genetic algorithm. Zhang et al. [8] focused the research on multi-criteria optimization of infrared bilberry drying. To find the Pareto-optimal value of drying time, consumed energy, rehydration capacity and ascorbic acid content, an artificial neural network was developed. Singh et al. [9] considered the analytical hierarchy method for the multi-criteria optimization of the microwave drying process of mango. Based on the analysis of the coefficients of a mathematical model, the minimum values of energy consumption and duration of drying were determined. Kiliç and Cinar [10] carried out an experimental study of the process of convective drying of vegetables. From the characteristics of the convective drying process of carrots, courgette and eggplant at various temperatures (60 °C, 70 °C and 80 °C), it could be seen that drying occurred during the period of falling rate of drying. An increase in the drying agent temperature from 60 °C to 80 °C decreased the drying time of carrot, courgette and eggplant by 35 %, 45 %, and 50 %, respectively, while the drying rate coefficients increased with increasing the drying agent temperature.

Wang and Rangaiah [11] highlighted that multi-criteria optimization provided several decisions that were equally good for achieving the set goal. These decisions are known as the Pareto-optimal frontier, which provides a deep insight into the choices of solutions to achieve the objectives. Selection of the optimal solution from a large number of possible ones is not a subject of much attention, although among the studied methods, the method of order of preference with a kind of the ideal solution is better suited for this purpose.

In order to evaluate the physico-chemical parameters of apples and apricots during drying in a pilot-production convective drying apparatus, the drying of cut fruits was observed at temperatures of 50 °C, 60 °C, 70 °C, 80 °C, air speed of 1.0 $m\cdot s^{-1}$, 1.5 $m\cdot s^{-1}$, 2.0 $m\cdot s^{-1}$ and relative humidity of 40–45 %. Veleşcu et al. [12] noted that, by increasing the air speed and temperature, the expected effect of water evaporation from the product was achieved. To study the effectiveness of the process of drying various fruits with a microwave oven, an experimental mathematical model was developed using the curve approximation method [13]. The model of the drying process was loaded into the smart oven processor and the drying process was controlled in real-time mode [14].

Vural et al. [15] considered multi-criteria optimal drying of olive leaves. In order to increase the oil extraction efficiency, the Pareto frontier was obtained from the predicted quadratic linear models using the least squares method. A compromise solution was found using the fuzzy c-means clustering algorithm (FCM). Winiczenko et al. [16] reported on the experimental research on the process of convective drying of apples in laboratory conditions. A mathematical model was obtained and optimization was carried out based on the pre-optimization criteria of apple colour difference, volume ratio and water absorption capacity.

Based on the analysis of the available scientific works on the topic presented above, it was found that various researchers were trying to achieve the efficiency of the fruits drying process by changing the temperature of the drying agent and its blowing speed, reducing the size of the product of a certain shape and increasing the drying surface area, using innovative and presented combined methods of drying, selecting the drying device, which the technical-economic indicators of the drying process depend on. With these measures, achieving the efficiency of the drying process of various fruits in industrial conditions is difficult and requires large economic costs. The existing management methods and automation software for convective drying of fruits cannot ensure the optimal management of the technological process, which is expressed in the achievement of minimum energy consumption during the drying process and the maximum possible productivity of the drying machine, while maintaining the standard amount (7 %) of residual moisture content of the product.

The requirement of the minimum energy consumption for the drying process and the maximum possible productivity of the drying machine simultaneously, under certain technological limitations, is considered among the classical problems of optimal control theory and can be solved by any of the methods proposed for this purpose. However, it is clear that the requirement to maintain the desired amount of residual moisture content of fruits and to achieve the maximum possible productivity of the drying unit are in conflict with each other, without taking into account the requirement of the minimum energy costs for drying. This task belongs to the scope of optimal control theory and its solution is implemented by an optimizing technique.

Based on the foregoing, it makes sense to set and solve the problem of optimal control of the fruits drying process with distributed parameters that occurs in the tunnel-type drying apparatus in static mode, as a multi-criteria (vector) optimization problem under technological constraints – by direct search for the saddle point of the linear fold of the individual optimal control criterion (with a Pareto-optimal solution [17, 18]), which is a scientific novelty of this research. The goal of this study was to develop an effective algorithm for managing the process of fruits drying in a tunnel-type drying machine, which would provide a solution to the multi-criteria optimal management task – minimizing energy costs during the drying process and achieving the maximum possible productivity of the tunnel-type drying machine, while maintaining the standard value of the residual moisture content of the product.

MATERIALS AND METHODS

Samples

To achieve the set goal, we selected and purchased ripe, dense apricots suitable for drying at the market in Kutaisi (Georgia) as the research object. Fruits of apricot (*Prunus armeniaca*) are a high-moisture product, whose dehydration and obtaining dried fruits is associated with considerable problems. The problems are caused by variability of properties and composition of fruits, depending on the cultivar, origin, growing conditions and the degree of ripeness. These circumstances predetermine the nutritional value and taste. In the study, the methods of convection heating, mathematical modelling, experimental research, vector optimization, non-linear programming and optimal control theory were used.

Equipment

Tunnel-type dehydrators are mostly used for drying stone fruits and grapes. Accordingly, in the development of the multi-criteria optimization algorithm of the static mode of the drying process, we used the technological data of the industrial tunnel-type dehydrator PR128T-3000/4 (Flight-M, Moscow, Russia) and applied the research results to it.

The drying unit was equipped with:

- automatic temperature measuring devices of the drying agent at the inlet and outlet of the drying chamber;
- automatic devices for measuring relative humidity of wet and exhaust air.

The dryer has an interface that works in parallel with the controller to realize the multi-criteria optimization algorithm of the static mode of drying in the tunnel-type device.

Methods

We measured the moisture content of apricots at the inlet and outlet of the drying chamber by laboratory analysis.

The product to be dried (e.g. fruits or vegetables) is laid in a single layer on grating plates, which are placed on a special drying trolley. The drying trolley in the dryer moves in the working chamber by means of a step-by-step conveyor. In the working chamber, the flow of the drying agent moves in the direction opposite to the movement of the drying trolley loaded with the drying product, gradually losing heat and gaining moisture, while the product moving, on the contrary, is heated and loses moisture.

Several drying trolleys are located in the chamber simultaneously, the number of which is determined by the design of the dryer. Periodically, at pre-determined intervals, one trolley with the dried product is unloaded at the outlet of the working chamber. Simultaneously, another trolley with the product to be dried is loaded at the inlet of the working chamber. The time intervals between loading of trolleys is determined by the estimated duration of the drying process for each type of the product being dried. During the time interval of drying, each trolley remains stationary in the working chamber.

To increase the efficiency of the tunnel-type dehydrator (quantified for 1 kg of evaporated moisture, depending on the required heat consumption), partial re-circulation of the working air can be used [1].

Technological control of the tunnel-type dehydrator is carried out according to the following positions:

- the temperature and relative humidity of the drying agent supplied to the outlet of the hot air dryer at the outlet of the working chamber;
- temperature and relative humidity of the drying agent released from the drying chamber and again supplied to the inlet of the drying chamber;
- the speed of air movement in the working chamber;
- pressure at the inlet and outlet of the chamber.

The tunnel-type drying equipment complicates the control of the temperature and humidity of the product during the drying process. Drying in tunnel-type dehydrators is carried out in batches, in a nearly stationary regime. Before launch, the drying agent, which is heated to a temperature of 80 °C, circulates inside the drying chamber. Once reached the required temperature in the drying chamber, the dryer is gradually loaded with a product and the established trolleys loading-unloading mode is maintained [1–3].

At the wet end of the channel, on the side of the loading position of trolleys, a temperature of 50–65 °C and 70–85 % relative humidity are usually maintained, while at the dry end of the channel (on the side of the unloading position of trolleys) temperature of 75–80 °C and 7 % relative humidity are usually maintained.

The final (residual) moisture content of dried fruits is regulated by the requirements of the standard [19] and its control is very important, since excess drying may lead to significant waste of heat and loss of product quality. On the other hand, insufficient drying may lead to microbiological instability of the product [3].

The main mode parameters of tunnel dehydrators are as follows:

- loading (stopping) period of the drying trolleys;
- air circulation and the location of the gates of a suction port;
- the location of the distribution valves ensuring even distribution of the drying agent flow in relation to the section of the drying channel;
- parameters of the drying agent at the bottom of the drying channel (temperature and humidity).

The main disturbing factors in the operation of tunnel dehydrator are as follows:

- changes in the product temperature and initial moisture content;
- changes in temperature and humidity of the incoming air;
- random change in the total amount of the product and non-uniformity of its distribution in the drying trolleys.

The change in the amount of fruits to be dried in the dryer and the non-uniformity of the product distribution are largely due to the manual loading technology of the drying trolleys, as well as the lack of means of mechanizing the loading position, as a result of which the technological process is unstable. For example, in order to equalize the residual moisture content in the volume of the product, after drying, it is often necessary to stop the drying trolleys in an enclosed area for a certain period of time.

We determined the initial moisture content of apricots according to GOST 32787-2014 [20]. The final moisture control was periodically carried measured by the gravimetric method according to GOST 9516-92 [21]. At the initial stage, we took on average 200 g sample of dried apricots and placed them in a drying cell separately. At the end of the drying cell, we determined the mass of the same sample. We determined the final moisture content based on the difference in the mass of the sample using an electronic digital analytical balance SF-400C model (Toms, Qilin, China) with a weighing accuracy of 0.01 g.

In general, the drying process in the tunnel-type dehydrators is a continuous-type process and corresponds to a nearly stationary mode. Since the operating mode affects the technological variables of the tunnel dryer, drying regulations and technology are not compatible. Therefore, the control of the process can only be based on optimization of the static mode [1–3].

Statistical analysis

Reliability of the obtained data was evaluated by T-test using the Windows IBM SPSS Statistics version 20.0 software (IBM, Armonk, New York, USA). To describe the ordered samples, we used statistical functions of the average arithmetic value and the average standard error. We used a reliability level of $p < 0.05$.

RESULTS AND DISCUSSION

In order to solve the problem of optimization of the static mode of drying of dried fruits in a tunnel dryer, three input (controlling) variables are needed, in particular:

$$u_1 = \tau_l \quad (1)$$

where u_1 is control variable 1 and τ_l is dwell time of the product in the drying chamber (in minutes);

$$u_2 = T_a(0) \quad (2)$$

where u_2 is control variable 2 and $T_a(0)$ is the drying agent temperature at the inlet of the drying chamber (in degrees Celsius);

$$u_3 = \beta \quad (3)$$

where u_3 is control variable 3 and β is the volume of fresh air sucked into the drying chamber from a drying agent preparation system (in cubic metres).

Since the goal of the research was the multi-criteria optimization of the static mode of the tunnel-type dehydrator with distributed parameters, we used a mathematical model of the tunnel-type dryer that we have already developed – a system of ordinary non-linear differential equations (Eq. 4–8 in Tab. 1), in the Cauchy normal form, which describes the distribution of the main technological parameters in the dryer along the longitudinal coordinate (l, in metres) of the dryer [3]. The product and air parameters at the outlet of the drying chamber can be determined as a solution of a system of differential equations (Eq. 4–8) describing the drying process which in a vector-matrix form can be more briefly written as follows:

$$y' = f(y, u), \quad y(0) = y_0 \quad (9)$$

The system Eq. 9 with boundary conditions ($0 \leq l \leq 1$) describes the distribution of stationary state values of the process variables along the longitudinal coordinate of the drying chamber linking the inlet and outlet of the drying chamber.

Since the wet air from the outlet of the dryer returns to its inlet, its moisture content at the inlet depends on the moisture content of the air at the outlet:

Tab. 1. Differential equations describing the drying process.

Technological parameters	At point l $y' = f(y,u)$	At inlet $y(0) = y_0$	Eq.
Pressure of drying agent [Pa]	$\frac{dy_1}{dl} = \frac{K_p}{\tau_a} = f_1(y,u)$	$y_1(0) = p_0$	4
Moisture content of wet product [%]	$\frac{dy_2}{dl} = -N(y) = f_2(y,u)$	$y_2(0) = w_0$	5
Temperature of product [°C]	$\frac{dy_3}{dl} = \frac{\alpha F(y_5 - y_3) - H_{vap} N(y)}{C_x} = f_3(y,u)$	$y_3(0) = T_{x_0}$	6
Moisture content of drying agent [%]	$\frac{dy_4}{dl} = \frac{K_a \tau_x}{\tau_a} N(y) = f_4(y,u)$	$y_4(0) = y_{a_0}$	7
Temperature of drying agent [°C]	$\frac{dy_5}{dl} = \frac{\alpha F(y_3 - y_5)}{C_a} = f_5(y,u)$	$y_5(0) = T_{a_0}$	8

l – longitudinal coordinate of the drying chamber (in metres), u – control variable.
Parameters at point l and at the inlet, respectively: $y_1(l)$, p_0 – pressure of the drying agent in the wet end of the drying chamber, on the side of loading the trolleys; $y_2(l)$, w_0 – moisture content of the wet product; $y_3(l)$, T_{x0} – temperature of the product to be dried; $y_4(l)$, y_{a0} – moisture content of the drying agent; $y_5(l)$, T_{a0} – temperature of drying agent.
αF – heat transfer coefficient between the drying agent and the product (in kilowatts per square metres per degrees Celsius); C_a – heat capacity of drying agent; C_x – heat capacity of the dried product; H_{vap} – specific evaporative heat of water (in kilojoules per kilogram); K_a – heat transfer coefficient between the drying agent and the product per unit mass of the dried product (in kilowatts per square metres per kilogram per degrees Celsius); K_p – heat transfer coefficient of the phase separation between the drying agent and the product per unit mass of the dried product (in kilowatts per square metres per kilogram per degrees Celsius); $N(y)$ – drying rate (in kilograms per minute); τ_a – time of movement of the drying agent in the drying chamber (in minutes); τ_x – time of product movement in the drying chamber (in minutes).

$$y_4(0) = \beta y_{env} + (1 - \beta) y_4(1) \quad (10)$$

$$G_F = K_H \left(\frac{i(0) - \beta \cdot i_{env}}{(1 - \beta)} - i(1) \right) \quad (11)$$

where β is volume of fresh air sucked into the dryer from the drying agent preparation system (in cubic metres); G_F is fuel consumption in the heat generator (in kilograms per hour); y_{env} is the moisture content of the air sucked into the drying chamber from the system of drying agent prepration (in percent); i_{env} is enthalpy of the air sucked into the drying chamber from the system of drying agent prepration (in kilojoules per mole); $i(0)$ and $i(1)$ is enthalpy of the wet air as a function of its temperature and moisture content ($i(0) = i(T_a(0), y(0))$, $i(1) = i(T_a(1), y(1))$, expressed in kilojoules per mole); K_H is proportionality factor between the added enthalpy and fuel consumption (in kilojoules per kilogram).

Thus, the ratio (Eq. 11) determines the fuel consumption necessary to maintain a constant temperature $T_a(0)$ at the inlet of the drying chamber. The temperature $T_a(0)$ can also be considered to have a controlling impact, then the ratio (Eq. 11) should be used to determine costs of heat for drying.

The technological scheme of the drying process in the tunnel-type installation is characterized by two major features [1, 3, 4]:

– use of both direct and reverse flow of the drying agent and the drying product;
– heating the drying agent released from the drying chamber and again supplied to the inlet of the drying chamber.

Although the direct flow technological scheme provides a smoother drying mode in the second period of drying, therefore, the frequent use of the reverse flow of the drying agent, in any case, leads to the need to consider the boundary problem of the static analysis of technological variables in the drying zone, which is provided by Eq. 10 and Eq. 11.

Based on the above, in order to determine the end value of components of the state vector $y(1)$ at the outlet of the dehydrator, when the input parameters $u = (u_1, u_2, u_3)$ change, it is necessary to simultaneously solve Eq. 9 and Eq. 10, that is, the analysis of each point of the static characteristic of the tunnel-type dehydrator, by means of the proposed mathematical model Eq. 9, will be reduced to the solution of the non-linear algebraic Eq. 10.

The drying process is characterized by two conflicting indicators of control quality and, in terms of the control quality assessment, the following requirements are imposed on the characteristics of the process [1, 5–7]:

Reducing of fuel consumption in a dehydrator to the minimum permissible value

$$J_1(u) = G_F \rightarrow \min \quad (12)$$

where $J_1(u)$ is criterion for fuel consumption.

The allowable minimum and maximum temperatures of the drying material at which the nutritional value of the product is maintained.

$$T_{min} \leq T(l) \leq T_{max}, \quad 0 \leq l \leq 1 \quad (13)$$

where T_{min} and T_{max} are given control limits for each type of the product being dried (in degrees Celsius). When controlling the tunnel dryer, within fairly large limits, we can also change holding time (τ_F) of the material in the drying chamber, which can be considered either as the product of the number of drying trolleys placed in the drying chamber and holding time, or as the ratio of the length of the drying chamber (l) to the average speed (ν) of the trolleys.

It is clear that holding time of the material in the dryer determines the productivity (P) of the dryer in terms of the amount of the dried material, which, together with the energy costs of the process, should be taken into account when selecting the optimal mode of the process. In this regard, it is appropriate to consider the tunnel dehydrator control problem to be a vector optimization problem with two types of quality criteria – criterion $J_1(u)$ in Eq. 12 and an additional criterion for productivity $J_2(u)$ in Eq. 14:

$$J_2(u) = P \rightarrow \max \quad (14)$$

Since the presented criteria Eq. 12 and Eq. 14 are contradictory, it is impossible to find the best solution. Therefore, a certain compromise is needed, which can be successfully found by the Pareto-optimization method [17].

Since drying is carried out until reaching a given value of the moisture content of the material that is being dried (w_F), for any of the above two criteria, the following condition should be kept additionally:

$$y_4(l) = w_F \quad (15)$$

Thus, the problems of energy cost minimization (Eq. 12) and drying plant production maximization (Eq. 14), in order to find the compromise between two requirements, under the constraints of Eq. 9, Eq. 10, Eq. 11, Eq. 13 and Eq. 15, are solved by the Pareto-optimal solution. The Pareto frontier is described as a parametric curve. For an effective and reliable approach to the Pareto frontier, the two-criteria optimization problem

Tab. 2. Dependence of the minimum value of a new criterion on the fold parameter.

Function	Equation	Eq.
Minimum value of the fold parameter	$J(\mu,u)=\mu\tau_l+(1-\mu)\left[\frac{\tau_l}{v_a}(1-\beta)(y_4(0)-T_{env})+\beta\left(1+K_{cp}y_5(1)\right)\left(y_4(0)-y_4(1)\right)\right]$	16

Fold parameter: $0 \le \mu \le 1$.

$J(\mu, u)$ – criterion for optimization of the dependence of the governing variables on the μ fold parameter, μ – fold parameter, u – governing variables, τ_l – time of movement of unit mass product (in minutes), v_a – speed of the drying agent movement in the drying chamber (in metres per minute), β – volume of fresh air sucked into the dryer (in cubic metres), $y_4(0)$ – moisture content of the drying agent at the entrance of the drying chamber (in percent), T_{env} – ambient temperature (in degrees Celsius); K_{cp} – ratio of heat capacities of air and water vapour, $y_5(1)$ – temperature of the drying agent at the outlet of the drying chamber (in degrees Celsius), $y_4(1)$ – moisture content of the drying agent at the outlet of the drying chamber (in percent).

was solved by the Runge-Kutta numerical method [18].

To that end, we introduced a linear fold of $J_1(u)$ and $J_2(u)$ criteria. Then, the dependence of the minimum value of a new criterion on the fold parameter (μ) was considered. In this case, the fold was written as Eq. 16 (Tab. 2).

To solve the set problem, it was necessary to solve the minimization problem of Eq. 16, under conditions of the restrictions of Eq. 13 and Eq. 15, and examine the dependence

$$J_{min}(\mu)=\min J(\mu,u) \quad (17)$$

within limits $0 \le \mu \le 1$ [22, 23].

The optimization problem was solved on an IBM-type computer using the Runge-Cutta method and the Matlab/Simulink (MathWork, Natick, Massachusetts, USA) dynamic systems modelling program Optimization Toolbox package [18, 24].

As a result of the study, it was found that the candidate points of the problem solution are the extremum points of the dependence Eq. 17. Fig. 1 illustrates the graph of the function Eq. 17, which shows that the function-criterion has two extreme values in the interval [0,1] – minimum μ_{min}, when $\mu = 0.307$, and maximum μ_{max}, when $\mu = 0.308$. An additional check of the fact that the point μ_{max} represented the solution of the minimization problem

$$J(0.308,u)\rightarrow \min \quad (18)$$

with the presence of the condition Eq. 15, was also the Pareto-optimal solution, for checking of which, there was no need for

$$J_1(u_{0.308}+\delta u)\le J_1(u_{0.308}) \quad (19)$$

$$J_2(u_{0.308}+\delta u)\le J_2(u_{0.308}) \quad (20)$$

where $u_{0.308}$ is the value of the control variable at the fold parameter $\mu = 0.308$, δu is growth of the governing variable.

The solution of the system of non-linear inequalities for a relatively small δu, and the solution of the Pareto-optimal problem, were directly realized by the Monte-Carlo method [25].

Once the Pareto-optimal set was found, the best decision was selected from the Pareto frontier domain [11]. The results of solving the tunnel dryer optimization problem obtained in this way, when drying of apricots with a stone, are presented in Tab. 3. It shows the compromise between the two requirements:

- J_1 – minimization of energy costs, which is satisfied by relatively low air temperature at the inlet of the dehydrator, at relatively low productivity;
- J_2 – maximization of productivity, which is achieved at the increase in energy costs, at a relatively high temperature of the incoming air.

Overall, optimal control of the tunnel-type dehydrator is carried out using the principle of disturbance control [1, 3, 4, 26].

Before starting the drying process, determine

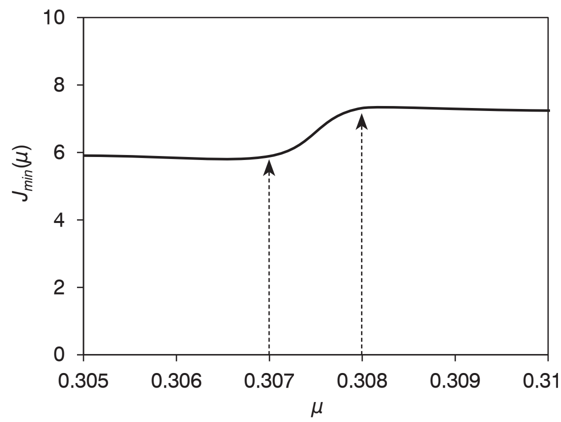

Fig. 1. Dependence of the minimum value of the fold criterion on parameter μ during dehydration of apricots with stones.

$J_{min}(\mu)$ – minimum value of the fold criterion; μ – fold parameter.

Tab. 3. The Pareto optimal frontier of the drying process of apricots with stones.

Control variable	Fold parameter μ	Control variable $\underline{u}$	Minimum point $J_1(u)$	Maximum point $J_2(u)$
Productivity of dehydrator [kg·h^{-1}]	0.307	1.533	1.322	1.467
	0.308	1.305	1.210	1.478
	0.310	1.333	1.472	1.910
Air temperature at the inlet of dehydrator [°C]	0.307	400.671	392.317	444.935
	0.308	381.306	391.751	445.367
	0.310	388.602	391.120	449.167
Share of the air sucked into the drying chamber from the drying agent preparation system [m^3]	0.307	0.669	0.261	0.359
	0.308	0.340	0.399	0.709
	0.310	0.352	0.537	0.712

$J_1(u)$ – criterion for fuel consumption, $J_2(u)$ – criterion for productivity.

the main characteristics of the product batch, which are necessary for designing the optimal program. These include type of the product, its initial moisture content, rate of loading of trolleys and atmospheric temperature).

These data are entered into the calculation device that implements the report of the optimal program (fuel costs in the heat generator, the amount of the outdoor air absorbed, time of heating the product loaded on the trolleys.

To ensure the stability and reliability of the multi-criteria optimization algorithm, we held testing aimed at confirming the expected functionality of the software modules and minimizing the probability of future failures. At the first stage, we checked the parts of the application that were independent of each other. All classes, functions and variables were tested. At the second stage, the testing was held from the beginning to the end, i.e. from the database to the user interface. All details were taken into account to make the two-criteria optimization process stable and reliable.

The multi-criteria optimal control program defined in this way was implemented in the drying process of a given batch of the product by means of an execution device.

For the fuel consumption generator of the tunnel-type dryer, after the optimal static mode is established, based on the control of the fuel consumption in the dryer, based on the control of the fuel consumption value G_F, and simple calculations, we determined a 5% reduction in the consumed energy compared to the state before optimization.

We assessed the quality of dried fruits using a 10-point scale according to organoleptic characteristics of appearance, shape, taste, odour and colour [19]. Dried apricots with optimal final moisture content of 7 %, obtained by using the multi-criteria optimization algorithm, were characterized by the best organoleptic indicators, compared to dried apricots with 12% moisture content. The shelf life was prolonged to 12 months at a storage temperature 0–10 °C and at relative humidity of no more than 75 %.

When implementing the research results obtained in this study, based on the calculation, the energy spent on drying decreased by 5 % and the quality of the dried product increased considerably.

CONCLUSIONS

The mathematical model with distributed parameters of the static mode along the longitudinal coordinate of the tunnel-type dryer developed in the study accurately reflects the real process, technological restrictions and goal criteria. It allowed us to find a Pareto-optimal solution for two-criteria optimization of the drying process using a modern computational approach. The stable enough and fast-acting algorithm for solving the problem of multi-criteria optimal management of the static mode of the tunnel-type drying process developed in the study allowed us to reduce energy costs by 5 % and improve the quality of the dried product. Dried apricots with a final moisture content of 7 % obtained as a result of the two-criteria optimization of the tunnel dryer were better in organoleptic indicators than dried apricots with a moisture content of 12 %. The formulated recommendations regarding the two-criteria optimal control algorithms and software implementation will help the industrial fruits processing technologists and managers in this field in the decision-making process.

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